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As to the marked difference of birds of the two kinds in relation to the condition of their bones, the rationale is not very obvious. Perhaps an approximation to the truth, or to the probable, may be made by comparing the bones of birds of the two kinds, which are possessed of similar powers, the swift, for instance, and the buzzard, rivals in swiftness of flight and enduring power of wing. How different are their humeri! of the former, very short, strong, and compact, provided with firm and large processes for the attachment of muscles; in the latter, long, hollow, and light, and comparatively brittle, yet sufficiently firm to bear without fracture the muscles which act on them. Here, have we not after a manner a kind of substitution of qualities? great strength and extended surface in small space in the one, for lightness with greater length of leverage in the other. Further, the one kind of bone, that which contains marrow, being less brittle than that which contains air, and more yielding, may be less liable to fracture; a quality which, in the bird, before the ossification is complete, may be of essential service; so that, teleologically considered, it may perhaps serve to account for the bones which are eventually hollow having primarily marrow in place of air.

December 13, 1866.

WILLIAM BOWMAN, Esq., Vice-President, in the Chair.

Among the Presents announced were two manuscript volumes, by Solomon Drach, Esq., F.R.A.S., containing various Tables in Pure Mathematics, presented by the author.

The following communications were read:—

I. "On Poisson's Solution of the Accurate Equations applicable to the Transmission of Sound through a Cylindrical Tube; and on the General Solution of Partial Differential Equations." By R. Moon, M.A., late Fellow of Queen's College, Cambridge. Communicated by Prof. J. J. Sylvester. Received November 14, 1866.

(Abstract.)

The pair of equations

$$\pm\frac{v}{a}=\log_{\varepsilon}\frac{\rho}{D},$$

$$v=\phi\{(v\pm a)t-q\},$$

which constitute Poisson's solution of the accurate equations applying to the transmission of sound through a cylindrical tube derived by La Grange's method, have long attracted the attention of mathematicians.

For La Grange's equations we may substitute the following, viz.—

$$\left.\begin{aligned}\frac{dv}{dt}+\frac{a^2}{D}\frac{d\rho}{dx}=0,\\ \frac{dv}{dx}+\frac{D}{\rho^2}\frac{d\rho}{dt}=0.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (A)$$

The first of these is obtained from the equation of the Encyc. Met. (Art. "Sound"), by putting v for $\frac{dy}{dt}$ and $\frac{D}{\rho}$ for $\frac{dy}{dx}$.

The second results from a similar substitution in the analytical condition,

$$\frac{d}{dx}\left(\frac{dy}{dt}\right)=\frac{d}{dt}\left(\frac{dy}{dx}\right).$$

Poisson's solution has always been regarded as imperfect, and may easily be shown to be so.

I was some time ago struck by finding that the above equations, while they yielded with facility the result of Poisson, notwithstanding their simplicity, baffled every effort to extract from them one more consonant to the general exigencies of the problem.

I had at this time arrived at the conclusion that the law of pressure assumed in the received theory of Fluid Motion could not be generally true, and in a Paper* communicated to the Royal Society, had pointed out that, in a certain case of motion, the assumption of the truth of that law led to a contradiction; while in another case of motion the expression for the pressure given by the received theory was palpably erroneous.

It occurred to me, therefore, that the defective law of pressure of the received theory accounted for the defective solution which alone was obtainable from the equations of motion derived from it. If the law of pressure of the received theory was not always true, if it held only when certain conditions were satisfied, those conditions would obviously have the effect of dismissing from the complete solution of the problem obtained on a perfect theory at least one of the two arbitrary functions which it must necessarily involve.

With a view to establishing this point, assume the solution of the equations of motion to be contained in the pair of equations,

$$\left.\begin{aligned}F(xyt\rho v)=\phi\{f(xyt\rho v)\},\\ \bar{F}(xyt\rho v)=\psi\{\bar{f}(xyt\rho v)\}.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (B)$$

If these equations satisfy the equations (A), the latter will equally be satisfied by the pair of equations,

$$F(xyt\rho v)=v,$$
$$\bar{F}(xyt\rho v)=\psi\{f(xyt\rho v)\}.$$

But it is shown in my Paper that these latter can only satisfy the equations (A) on the supposition that F is of the form

$$F=F(v\pm a\log_{\epsilon}\rho).$$

Moreover, since we have in equations (B) the function F equal to an

* On the True Theory of Pressure as applied to Elastic Fluids in Motion.

arbitrary function of f, conversely we must have f equal to an arbitrary function of F. Hence, in exactly the same way in which it is proved that F must have the above form, we may equally prove that f, F, f must severally have the same form. It clearly results, therefore, from the above assumed solution (B), that the equations (A) are insoluble, except on the assumption that the velocity may be expressed in terms of the density alone; *i. e.* that we may assume

$$v=f(\rho).$$

But substituting this value of v in equations (A), the latter become

$$f'(\rho)\frac{d\rho}{dt}+\frac{a^2}{\mathrm{D}}\frac{d\rho}{dx}=0,$$

$$f'(\rho)\frac{d\rho}{dx}+\frac{\mathrm{D}}{\rho^2}\frac{d\rho}{dt}=0,$$

whence we get

$$\overline{f'(\rho)}\Big|^2=\frac{a^2}{\rho^2}, \qquad \overline{\frac{d\rho}{dt}}\Big|^2=\frac{a^2\rho^2}{\mathrm{D}^2}\overline{\frac{d\rho}{da}}\Big|^2;$$

and eventually, D being the value of ρ when $v=0$,

$$\pm\frac{v}{a}=\log\,.\,\frac{\rho}{\mathrm{D}},$$

$$\rho=\phi\left\{x\mp\frac{a\rho}{\mathrm{D}}.t\right\},$$

which is the most general solution of which the equations of motion are susceptible; and which, making allowance for the difference of the ordinates employed in the two cases, y referring to the particle when in motion, and x to its position of rest, is identical with that of Poisson.

The failure of mathematicians to derive from the equations of motion of the received theory a solution containing two arbitrary functions has hitherto, I apprehend, been universally attributed to difficulties of integration. So far is this view from being well founded, however, that in a postscript to my Paper it is shown that, *assuming the pressure to follow any law whatever*, a solution of the equations of motion can be obtained containing two arbitrary functions; a result, however, which requires that the expression for the pressure shall satisfy certain conditions, which conditions are *violated* when the pressure is assumed to vary with the density alone.

Whatever be the law of pressure, it must always be capable of being expressed in terms of x and t. Moreover, the velocity and density are in like manner severally capable of being expressed in terms of x and t; whence it follows that we may always express x in terms of ρ and v, and equally that we can express t in terms of ρ and v; so that, whatever be the law of pressure, we may assume it to be a function of ρ and v.

Hence, assuming

$$\frac{dp}{dx}=\frac{dp}{d\rho}\cdot\frac{d\rho}{dx}+\frac{dp}{dv}\cdot\frac{dv}{dx}=\mathrm{R}\frac{d\rho}{dx}+\mathrm{V}\frac{dv}{dx},$$

where R and V are functions of ρ and v only, we have for our equations of motion the following, viz.

$$0=\frac{dv}{dt}+\frac{R}{D}\frac{d\rho}{dx}+\frac{V}{D}\frac{dv}{dx},$$

$$0=\frac{dv}{dx}+\frac{D}{\rho^2}\frac{d\rho}{dt},$$

of which the following pair of equations constitute the solution, viz.

$$f_1(\rho,v)=\phi\left\{x-\frac{V+\sqrt{V^2+4R\rho^2}}{2D}.t\right\},$$

$$f_2(\rho,v)=\psi\left\{x-\frac{V-\sqrt{V^2+4R\rho^2}}{2D}.t\right\}.$$

where $f_1(\rho,v)=$const., and $f_2(\rho,v)=$const. are the respective integrals of the ordinary differential equations,

$$dv-\frac{V-\sqrt{V^2+4R\rho^2}}{2\rho^2}d\rho=0,$$

$$dv-\frac{V+\sqrt{V^2+4R\rho^2}}{2\rho^2}d\rho=0,$$

which involve the variables v and ρ only. But this result is dependent on the fact of the following conditions being satisfied, viz., that we have

$$K_1\frac{dK_1}{d\rho}+R\frac{dK_1}{dv}=0,$$

$$K_2\frac{dK_2}{d\rho}+R\frac{dK_2}{dv}=0,$$

where

$$K_1=V-\sqrt{V^2+4R\rho^2},$$

$$K_2=V+\sqrt{V^2+4R\rho^2},$$

which conditions *cannot* be satisfied if the law of pressure depends upon the density only (in which case $V=0$ and R contains ρ only), as may easily be shown.

With regard to the theory of Partial Differential Equations, I conceive that the methods indicated in the Paper will serve to elicit *every* solution of a partial differential equation of the second order and first degree, save one, viz., an integral solution consisting of a simple relation between the variables x, y, z, free from arbitrary functions, and which is not derivable from a solution containing arbitrary functions, by assigning particular values to the latter.

If the equation

$$0=R\frac{d^2z}{dx^2}+S\frac{d^2z}{dxdy}+T\frac{d^2z}{dy^2}-V$$

have a first integral consisting of the pair of equations

$$F(xyzpq)=\phi\{f(xyzpq)\},$$

$$\bar{F}(xyzpq)=\psi\{\bar{f}(xyzpq)\},$$

or consisting of the pair of equations

$$F(xyzpq)=0,$$
$$\bar{F}(xyzpq)=\psi\{\bar{f}(xyzpq)\},$$

or of the single equation

$$F(xyzpq)=\phi\{f(xyzpq)\},$$

or of the single equation

$$F(xyzpq)=0,$$

then in every one of these cases we must have

$$0=R\,.\,\overline{F'(q)}|^2-S\,.\,F'(q)\,.\,F'(p)+T\,.\,\overline{F'(p)}|^2,$$
$$0=R\,.\,F'(x)\,.\,F'(q)+T\,.\,F'(p)\,.\,F'(q)+\{R\,.\,F'(q)\,.\,p+T\,.\,F'(p)\,.\,q\}F'(z)$$
$$+V\,.\,F'(p)\,.\,F'(q)\text{*}.$$

It is also shown with more or less of generality, and it is capable of being shown generally, that if the given equation admit of a complete integral solution containing two arbitrary functions, it will necessarily have two first integrals, each of which will be of the form

$$F(xyzpq)=\phi\{f(xyzpq\}.$$

It might have been added, that if the general equation of the second order and second degree be written

$$0=P_{r_2}\,.\,r^2+P_{s_2}\,.\,s^2+P_{t_2}\cdot t^2+P_{rs}rs+P_{rt}\,.\,rt+P_{st}\,.\,st$$
$$+P_r r+P_s\,.\,s+P_t+P;$$

and it is satisfied by the equation

$$F(xyzpq)=\phi\{f(xyzpq)\},$$

then F and f must severally satisfy *all three* of the following equations, viz.

$$F'(p)-l\,F'(q)=0,$$
$$F'(x)+F'(z)p-mF'(p)=0,$$
$$F'(y)+F'(z)q-nF'(q)=0,$$

where

$$0=P_{t_2}\,.\,l^4-P_{st}\,.\,l^3+(P_{s_2}+P_{rt})l^2-P_{rs}\,.\,l+P_{r_2},$$
$$0=P_{r_2}m^2+P_{rt}mn+P_{t_2}n^2-P_r m-P_t n+P,$$
$$0=(2P_{r_2}-P_{rs}\,.\,l+P_{rt}\,.\,l^2)m$$
$$+(2P_{t_2}l^2-P_{st}\,.\,l+P_{rt})n$$
$$-(P_t l^2-P_s l+P_r).$$

* The functions f, $\bar{F}$, $\bar{f}$ must satisfy the same conditions, except in the second of the above cases.